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Imaginary phases in two-level model with spontaneous decay

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Abstract

We study a two-level model coupled to the electromagnetic vacuum and to an external classic electric field with fixed frequency. The amplitude of the external electric field is supposed to vary very slowly in time. Garrison and Wright (1988 *Phys. Lett.* A **128** 177) used the non-Hermitian Hamiltonian approach to study the adiabatic limit of this model and obtained that the probability of this two-level system to be in its upper level has an imaginary geometric phase. Using the master equation for describing the time evolution of the two-level system we obtain that the imaginary phase due to dissipative effects is time-dependent, in opposition to the Garrison and Wright result. The present results show that the non-Hermitian Hamiltonian method should not be used to discuss the nature of the imaginary phases in open systems.

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1. Introduction

The natural approach to study quantum systems in contact with an environment is the density operator method. It allows one to study quantum states evolving from pure to mixed states. In general, exponential decay in the time evolution of the population at each excited state of the quantum system occurs due to the exchange between the system and its environment. An alternative approach to obtain those exponential decays in the probability is through complex energies obtained from a phenomenological non-Hermitian Hamiltonian [1–4]. The non-Hermitian Hamiltonian method has been very fruitful in approaching various physical problems, for example, the multiphoton ionization [2–5] and free-electron laser theory [6, 7]. As mentioned by Baker in [3], it is expected that the non-Hermitian Schrödinger equation be a *bona fide* description of the interaction among the parts of a system when the intervals of time are sufficiently short such that the coherence state of a subsystem is not destroyed by its interaction with the environment.

Since the disclosure of geometrical phases by Berry in 1984 [8] in cyclic Hamiltonians evolving adiabatically, there has been a significant search for geometrical phases in other physical contexts. For example, Joye *et al* [9] and Berry [10] independently showed that the transition probability of instantaneous eigenstates of non-real Hamiltonians in the non-adiabatic regime has an imaginary geometric phase. This imaginary geometric phase was measured by Zwanziger *et al* in a two-level system [11]. A great part of the work done on geometric phases has been on pure states. Since the work by Uhlmann in 1986 [12, 13], however, the study of holonomy has been extended to mixed states under unitary evolution [14–16]. More recently, Ericsson *et al* [17] obtained the expression for the geometric phase of a quantum system interacting with its environment when the unitary evolution of the whole system (including the environment) is known. Certainly the discussion of holonomy in mixed states is a very interesting point, since the correct expression of the geometric phase for mixed states under unitary evolution is still under debate, as in a very recent work by Singh *et al* [18]. However, that is not the issue of the present communication. Another equally interesting question about the imaginary phases in transition probabilities of open systems is the correctness of the application of the non-Hermitian Hamiltonian approach [19–22] to discuss the nature of such phases.

In particular, Garrison and Wright [19] used the non-Hermitian Hamiltonian method to study a two-level model with linewidths in the presence of an external electric field, obtaining the adiabatic limit of the probability of this system being in its upper level, after the external field has returned to its original configuration. They concluded that the decaying factor has an imaginary correction to Berry's phases. Their result for the two-level model coupled to an external classical electromagnetic field with spontaneous emissions is already contained in the phases (75) and (76) of [3] for any open system described by a non-Hermitian Hamiltonian. It is the mathematical structure of these phases in the non-Hermitian Hamiltonian approach that makes the imaginary phase derived by Garrison and Wright in [19] have an imaginary geometric contribution.

Garrison and Wright mentioned in their conclusion to [19] that the nature (time- or path-dependent) of their imaginary phase due to dissipation effects should be re-examined using the density matrix approach; this is the aim of the present letter.

In [23] we studied the adiabatic limit of any periodic non-degenerate Hamiltonian using the density matrix approach (extending the discussion carried out by Born and Fock in [24] to the density matrix on the basis of instantaneous eigenstates of the Hamiltonian), and we concluded that for a quantum system to have an imaginary correction to Berry's phases in dissipative phenomena, the functions in the integrals of the decay exponentials in the entries of the density matrix would have to satisfy special conditions. Let $\mathrm{e}^{-\int_0^t \mathrm{d}t' c(t')}$ be a typical decreasing exponential originating from the presence of dissipative effects. For the integral in the exponential to be written as a path-dependent integral, the function $c(t)$ must have the form

$$c(t) = \varphi_i(t)\frac{\mathrm{d}}{\mathrm{d}t}(\Psi_i(t)) \tag{1}$$

and the functions $\Psi_i(t)$ have to satisfy two conditions: (i) they must not be explicitly time-dependent; (ii) the time-dependence of functions $\Psi_i(t)$ must come only from their dependence on the set of parameters $\vec{\mathbf{k}}(t) \equiv (k_1(t), k_2(t), \ldots, k_l(t))$. We point out that there is no restriction to the regime of the time variation of the set of parameters $\vec{\mathbf{k}}(t)$ and that it must not be a periodic function in time, i.e., the path in the $\vec{\mathbf{k}}$-parameter space must not be closed (see details in [23]).

In [19], Garrison and Wright considered a two-level system interacting with a classic external electromagnetic field

$$\vec{E}_{\text{clas}}(t) = \text{Re}[\vec{e}\mathcal{E}(t)\,\text{e}^{\text{i}\nu t}] \tag{2a}$$

where the amplitude $\mathcal{E}(t)$ varies very slowly. The two energy states were supposed to have a linewidth. They used the time-dependent non-Hermitian Schrödinger (Bethe–Lamb) equation in the rotating wave approximation (RWA)[19, 25]

$$\text{i}\frac{\text{d}}{\text{d}t}\begin{pmatrix} C_a(t) \\ C_b(t) \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}\text{i}\gamma_a & V^*\text{e}^{\text{i}\Delta t} \\ V\text{e}^{-\text{i}\Delta t} & -\frac{1}{2}\text{i}\gamma_b \end{pmatrix}\begin{pmatrix} C_a(t) \\ C_b(t) \end{pmatrix} \tag{2b}$$

where γ_a and γ_b are the decay rates for the upper and lower levels, respectively, and $C_a(t)$ and $C_b(t)$ are the corresponding probability amplitudes of states $|a\rangle$ and $|b\rangle$. Moreover, $\Delta = \omega - \nu$, $\omega = (E_a - E_b)/\hbar$, $V = \vec{\mu}\cdot\vec{e}\mathcal{E}(t)/2\hbar$ and $\vec{\mu} = \vec{\mu}_{ab}$ is the electric dipole matrix element. Letting $\mathcal{E}(t) = \mathcal{E}_0\,\text{e}^{\text{i}\phi(t)}$, where $\phi(0) = 0$ and $\phi(T) = 2\pi$, with $T \gg \frac{2\pi\hbar}{E_a - E_b}$ they obtained a complex Berry's phase β_- in the expression for the probability of the quantum system to be in the state a at time T,

$$\beta_- = \frac{1}{2}[2\pi(1 - \cos(\theta_0))] = \frac{1}{2}\left[2\pi\left(1 - \frac{\Delta - \text{i}\delta}{\sqrt{|2V_0|^2 + (\Delta - \text{i}\delta)^2}}\right)\right] \tag{2c}$$

where $\delta \equiv (\gamma_a - \gamma_b)/2$ and $V_0 = \vec{\mu}\cdot\vec{e}\mathcal{E}_0/2\hbar$.

In the present work we want to verify if the result on the path-dependence of the imaginary phases derived by the non-Hermitian method is faithful. To do so, we apply the density matrix approach presented in [23] to study the same physical problem considered in [19] with $\gamma_b = 0$. We wish to compare the nature (path- or time-dependent) of the imaginary phases in the expression of the probability of the atomic electron, initially in state $|a\rangle$, being in the same state at time $t = T$, derived by the density matrix approach with the one obtained in the non-Hermitian phenomenological Hamiltonian framework.

Let the two-level system that describes an atomic electron be represented by the Hamiltonian $\mathbf{H}_e$. The atomic electron interacts with a classic external electromagnetic field $\vec{E}_{\text{clas}}(t)$ (see equation (2*a*)) and with the electromagnetic vacuum (the vacuum electromagnetic field operator being represented by $\vec{\mathbf{E}}_0(\vec{x})$). The total Hamiltonian for this model in the Schrödinger picture is [26]

$$\mathbf{H}_T = \mathbf{H}_e + \mathbf{H}_f + \mathbf{H}_{\text{int}} \tag{3}$$

with

$$\mathbf{H}_e = \frac{\tilde{\mathbf{P}}^2}{2m} + V(r) \tag{4a}$$

$$\mathbf{H}_f = \sum_{\vec{k}}\sum_{\lambda=1}^{2}\hbar\omega_{\vec{k}}\left(\mathbf{a}^\dagger_{\vec{k}\lambda}\mathbf{a}_{\vec{k}\lambda} + \frac{1}{2}\right) \tag{4b}$$

and

$$\mathbf{H}_{\text{int}} = -\text{e}(\vec{E}_{\text{clas}}(t) + \vec{\mathbf{E}}_0(\vec{x}))\cdot\vec{\mathbf{r}} \tag{4c}$$

where $\tilde{\mathbf{P}}$ is the momentum operator associated with the atomic electron, $V(r)$ is the spherical interaction potential between the electron and the rest of atom and $\mathbf{H}_f$ is the Hamiltonian of the electromagnetic energy operator of the electromagnetic vacuum. The operator $\mathbf{a}^\dagger_{\vec{k}\lambda}$ ($\mathbf{a}_{\vec{k}\lambda}$) creates (destroys) a photon with momentum $\vec{k}$ in the polarization state λ. We also have: $\omega_{\vec{k}} = c|\vec{k}|$.

Following [19], the classic external electromagnetic field $\vec{E}_{\rm clas}(t)$ is given by equation (2*a*) assuming that $\mathcal{E}(t)$ varies very slowly and $\vec{e}$ is constant. The electromagnetic field operators in the vacuum $\vec{\mathbf{E}}_0(\vec{x})$ in the Schrödinger picture have the expansion

$$\vec{\mathbf{E}}_0(\vec{x}) = \sum_{\vec{k}} \sum_{\lambda=1}^{2} \vec{\epsilon}_{\vec{k}\lambda} \sqrt{\frac{\hbar\omega_{\vec{k}}}{2\epsilon_0 V}} \left(-\mathrm{i}\mathbf{a}^{\dagger}_{\vec{k}\lambda} \mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}} + \mathrm{i}\mathbf{a}_{\vec{k}\lambda} \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}}\right). \tag{4d}$$

The dynamics of the density operator of the complete system is driven by the Liouville von Neumann equation. Taking the trace over the electromagnetic degrees of freedom we obtain the master equation for the reduced density matrix of the atomic electron $\bar{\rho}(t)$ written on the basis of the eigenstates of $\mathbf{H}_e$ ($\mathbf{H}_e|\mathrm{i}\rangle = E_i|\mathrm{i}\rangle$). The dynamics of $\bar{\rho}(t)$ in the electric dipole approximation and in the RWA is [27, 28]

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\bar{\rho}(t) = {} & \frac{1}{\mathrm{i}}[(\omega_0 + \Omega_+)\sigma_z, \bar{\rho}(t)] - \frac{1}{\mathrm{i}\hbar}\vec{\mu}_{ab}\cdot\vec{E}_{\rm clas}(t)[\sigma_x, \Lambda^0(t)] + \gamma(2\sigma_-\bar{\rho}(t)\sigma_+ - \{\sigma_+\sigma_-, \bar{\rho}(t)\}) \\ & + \frac{\vec{\mu}_{ab}\cdot\vec{E}_{\rm clas}(t)}{\hbar^2}(-2A(t)\bar{\rho}(t) + 2A(t)\sigma_x\bar{\rho}(t)\sigma_x + B(t)[[\sigma_y, \bar{\rho}(t)], \sigma_x])\end{aligned} \tag{5}$$

where σ_z and $\sigma_\pm$ are the Pauli matrices with $\sigma_\pm = \frac{1}{2}(\sigma_x \pm \mathrm{i}\sigma_y)$, $\vec{\mu}_{ab}$ is the electric dipole matrix (we are supposing $\vec{\mu}_{ab}$ to be real) with $\vec{\mu}_{ab} \equiv e\langle a|\vec{\mathbf{r}}|b\rangle (e > 0)$,

$$\gamma = \frac{\pi}{\hbar^2}\sum_{\lambda=1}^{2}\int \mathrm{d}^3\vec{k}\,\eta(\vec{k})|g(\vec{k},\lambda)|^2\delta(\omega_0 - \omega_{\vec{k}}) \tag{6a}$$

being $g(\vec{k},\lambda) = -\mathrm{i}\vec{\epsilon}(\vec{k},\lambda)\cdot\vec{\mu}_{ab}\sqrt{\frac{\hbar\omega_{\vec{k}}}{2\epsilon_0 V}}$ and $\eta(\vec{k})$ is the density of states introduced in the integration,

$$\omega_0 \equiv \frac{E_a - E_b}{2\hbar} = \frac{\omega}{2} \tag{6b}$$

$$\Lambda^0_{ij}(t) \equiv \mathrm{e}^{-\frac{\mathrm{i}}{\hbar}(E_i - E_j)t}\rho_{ij}(0) \qquad i, j = a, b \tag{6c}$$

and

$$\Omega_+ = -\frac{\gamma}{\pi}\,\ln\left[\left|\frac{\omega_c}{\omega_0} - 1\right|\left(\frac{\omega_c}{\omega_0} + 1\right)\right] \tag{6d}$$

where ω_c is the cutoff frequency that preserves the dipole approximation ($\omega_c < c/a_0$, with a_0 the atomic Bohr radius). The cutoff ω_c becomes a parameter of the effective model to be determined by fitting to experimental data. The term Ω_+ corresponds to the frequency shift [27]. The elements $\rho_{ij}(0)$ are the initial values of the entries of matrix $\bar{\rho}(t)$. The functions $A(t)$ and $B(t)$ appearing in equation (5) are defined as: $A(t) \equiv \int_0^t \mathrm{d}t'\vec{\mu}_{ab}\cdot\vec{E}_{\rm clas}(t')\cos\left[\frac{(E_a - E_b)}{\hbar}(t - t')\right]$ and $B(t) \equiv \int_0^t \mathrm{d}t'\vec{\mu}_{ab}\cdot\vec{E}_{\rm clas}(t')\sin\left[\frac{(E_a - E_b)}{\hbar}(t - t')\right]$.

We recall that the density matrix of a two-level model must satisfy two conditions: (i) $\mathrm{Tr}(\bar{\rho}(t)) = 1$ and (ii) $\rho_{ba}(t) = (\rho_{ab}(t))^*$, where $\rho_{ab}(t) \equiv \langle a|\bar{\rho}(t)|b\rangle$. As a consequence of those conditions, the density matrix has only two independent elements; we choose $\rho_{aa}(t)$ and $\rho_{ab}(t)$ to be such elements. From equation (5) the time equations for those two elements are

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\rho_{aa}(t) = {} & [-2\gamma - 4\vec{\mu}_{ab}\cdot\vec{E}_{\rm clas}(t)\mathrm{Re}(\mathrm{e}^{\mathrm{i}(E_a - E_b)t}G(t))]\rho_{aa}(t) \\ & + 2\vec{\mu}_{ab}\cdot\vec{E}_{\rm clas}(t)\mathrm{Im}[\mathrm{e}^{-\mathrm{i}(E_a - E_b)t}\rho_{ab}(0)] + 2\vec{\mu}_{ab}\cdot\vec{E}_{\rm clas}(t)\mathrm{Re}[\mathrm{e}^{\mathrm{i}(E_a - E_b)t}G(t)]\end{aligned} \tag{7a}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{ab}(t) = (-2\mathrm{i}(\omega_0+\Omega_+) - \gamma - 2\vec{\mu}_{ab}\cdot\vec{E}_{\mathrm{clas}}(t)G(t)\,\mathrm{e}^{\mathrm{i}(E_a-E_b)t})\rho_{ab}(t) + 2\vec{\mu}_{ab}\cdot\vec{E}_{\mathrm{clas}}(t)G^*(t)\,\mathrm{e}^{-\mathrm{i}(E_a-E_b)t}\rho_{ba}(t) + \mathrm{i}\vec{\mu}_{ab}\cdot\vec{E}_{\mathrm{clas}}(t)(1-2\rho_{aa}(0)). \quad (7b)$$

The expression of the function $G(t)$ is: $G(t) \equiv \int_0^t \mathrm{d}t'\vec{\mu}_{ab}\cdot\vec{E}_{\mathrm{clas}}(t')\,\mathrm{e}^{-\mathrm{i}(E_a-E_b)t'}$. We see from equation (7*b*) that the real and imaginary parts of $\rho_{ab}(t)$ are coupled due to non-linear effects in the classic external electromagnetic field. From equations (7) onward, we will be using natural units ($\hbar = c = 1$).

In the same manner as in [23, 29], in order to study the $T \to \infty$ limit of equations (7) we apply the transformation

$$\tilde{\rho}_{ij}(t) \equiv \mathrm{e}^{\mathrm{i}(E_i-E_j)t}\rho_{ij}(t) \qquad i,j = a,b \quad (8)$$

and change the time scale to $s = t/T$; in the limit $T \to \infty$, one obtains

$$\frac{\mathrm{d}}{\mathrm{d}s}\tilde{\rho}_{aa}(s) \approx T[-2\gamma - 2\vec{\mu}_{ab}\cdot\vec{e}\mathcal{E}_0\mathrm{Re}(\tilde{G}(s)\,\mathrm{e}^{-\mathrm{i}(\phi(s)-\Delta Ts)})]\tilde{\rho}_{aa}(s) + T\vec{\mu}_{ab}\cdot\vec{e}\mathcal{E}_0[\mathrm{Im}(\mathrm{e}^{\mathrm{i}(\phi(s)-\Delta Ts)}\tilde{\rho}_{ab}(0)) + \mathrm{Re}(\tilde{G}(s)\,\mathrm{e}^{-\mathrm{i}(\phi(s)-\Delta Ts)})] \quad (9a)$$

and

$$\frac{\mathrm{d}}{\mathrm{d}s}\tilde{\rho}_{ab}(s) \approx -(2\mathrm{i}\Omega_+ + \gamma + \vec{\mu}_{ab}\cdot\vec{e}\mathcal{E}_0\tilde{G}(s)\,\mathrm{e}^{-\mathrm{i}(\phi(s)-\Delta Ts)})T\tilde{\rho}_{ab}(s) + \vec{\mu}_{ab}\cdot\vec{e}\mathcal{E}_0\tilde{G}^*(s)\,\mathrm{e}^{-i(\phi(s)-\Delta Ts)}T\tilde{\rho}_{ba}(s) + \mathrm{i}T\vec{\mu}_{ab}\cdot\vec{e}\frac{\mathcal{E}_0}{2}(1-2\tilde{\rho}_{aa}(0))\,\mathrm{e}^{-\mathrm{i}(\phi(s)-\Delta Ts)}. \quad (9b)$$

As we write equations (9), we are assuming that $\Delta \equiv 2\omega_0 - \nu \sim \frac{2\pi}{T}$. The terms proportional to $\mathrm{e}^{\pm\mathrm{i}((2\omega_0+\nu)Ts+\phi(s))}$ do not contribute in the limit $T \to \infty$ (as shown in [23, 29]) and the terms proportional to $\mathrm{e}^{\pm\mathrm{i}(\Delta Ts-\phi(s))}$ contribute to the dynamics of the density matrix in the resonance region when $\Delta \sim \frac{2\pi}{T}$. The function $\tilde{G}(s)$ appearing in equations (9) is given by $\tilde{G}(s) = \vec{\mu}_{ab}\cdot\vec{e}\frac{\mathcal{E}_0 T}{2}\int_0^{sT}\mathrm{d}s'\mathrm{e}^{\mathrm{i}(\phi(s')-\Delta Ts')}$.

Equation (9*a*) gives us the probability that the atomic electron is in state $|a\rangle$. Its solution after one period T is

$$\rho_{aa}(T) = \left\{\rho_{aa}(0) + \mathrm{i}\int_0^T \mathrm{d}t'\left(\frac{\mathrm{d}\tilde{G}(t')^*}{\mathrm{d}t'}\tilde{\rho}_{ba}(0) - \frac{\mathrm{d}\tilde{G}(t')}{\mathrm{d}t'}\tilde{\rho}_{ab}(0)\right)\mathrm{e}^{2(\gamma t'+|\tilde{G}(t')|^2)} + \frac{1}{2}\int_0^T \mathrm{d}t'\mathrm{e}^{2\gamma t'}\frac{\mathrm{d}[\mathrm{e}^{2|\tilde{G}(t')|^2}]}{\mathrm{d}t'}\right\}\mathrm{e}^{-2(\gamma T+|\tilde{G}(T)|^2)}. \quad (10)$$

Since the function $\tilde{G}(t)$ is an explicit time-dependent function, the condition *i* in equation (1) is not satisfied and none of the integrals in equation (10) is a time-independent integral. We have an overall exponential decay (imaginary factor), but from the definition of the constant γ (see equation (6*a*)), we obtain that the exponential $\mathrm{e}^{-2\gamma t}$ is not a geometric (path-dependent) imaginary phase, but a time-dependent one as well as the contribution to the decreasing exponential coming from $|\tilde{G}(T)|^2$.

For the sake of completeness, we should also examine the solution of equation (9*b*) after a period T. Differently from [29], here the dynamical equation of $\rho_{ab}(t)$ couples its real and imaginary parts under the regime of a strong classic external electric field. We get a $SU(2)$ structure for the solution of $\rho_{ab}(t)$. Calling $\rho_1(t) = \rho_{ab}(t)$ and $\rho_2(t) = \rho_{ba}(t)$, the solution of

equation (9*b*) is

$$\rho_I(T) = \left[\mathcal{T}\left(\mathrm{e}^{\int_0^T \mathrm{d}t'\vec{\mathcal{B}}(t')\cdot\vec{\sigma}}\right)\right]_{IJ}\left\{\int_0^T \mathrm{d}t'\left[\mathcal{T}\left(\mathrm{e}^{\int_T^0 \mathrm{d}t''\vec{\mathcal{B}}(t'')\cdot\vec{\sigma}}\right)\right]_{JK} \times \mathrm{e}^{(\gamma t'+\mathrm{Re}^2(\tilde{G}(t'))-\mathrm{Im}^2(\tilde{G}(t')))}d_K(t') + \rho_J(0)\right\}\mathrm{e}^{(-1)^I 2\mathrm{i}\omega_0 T}\,\mathrm{e}^{-(\gamma T+\mathrm{Re}^2(\tilde{G}(T))-\mathrm{Im}^2(\tilde{G}(T)))} \tag{11}$$

with $I = 1, 2$ and $J, K = 1, 2$. In these two last indices we are using the implicit sum notation on the rhs of equation (11). The symbol $\mathcal{T}$ means the time-ordering integrals[3], and $\vec{\sigma}$ are the Pauli matrices. The elements of the column $d_J(t)$ are: $d_1(t) = d_2^*(t) = \mathrm{i}\vec{\mu}_{ab}\cdot\vec{e}\frac{\mathcal{E}_0}{2}[1-2\rho_{aa}(0)]\,\mathrm{e}^{-\mathrm{i}(\phi(t)-\Delta t)}$. The components of the vector $\vec{\mathcal{B}}$ are: $\mathcal{B}_x(t) = \frac{\mathrm{d}[|\tilde{G}(t)|^2]}{\mathrm{d}t}$, $\mathcal{B}_y(t) = 2\left[\mathrm{Im}(\tilde{G}(t))\frac{\mathrm{d}(\mathrm{Re}(\tilde{G}(t)))}{\mathrm{d}t} - \mathrm{Re}(\tilde{G}(t))\frac{\mathrm{d}(\mathrm{Im}(\tilde{G}(t)))}{\mathrm{d}t}\right]$ and $\mathcal{B}_z(t) = -2\mathrm{i}\left[\Omega_+ + \frac{\mathrm{d}}{\mathrm{d}t}[\mathrm{Re}(\tilde{G}(t))\mathrm{Im}(\tilde{G}(t))]\right]$. We have again the function $\tilde{G}(t)$ that is explicity time-dependent and then the time-ordering integrals cannot be converted to path-ordering integrals. Therefore, the integrals and the phases on the rhs of equation (11) are all time-dependent, as well as the functions with the overall decreasing exponential.

In [19] Garrison and Wright present in equation (4.9) the value of the geometric imaginary phase in the limit of weak electric field and $\gamma_b \gg \gamma_a$. In the weak electric field limit, the $SU(2)$ structure in equation (11) disappears and the solutions of equations (10) and (11) become simpler,

$$\rho_{aa}(t) = \left\{\rho_{aa}(0) + \mathrm{i}\int_0^t \mathrm{d}t'\left(\frac{\mathcal{E}_0}{2}\vec{\mu}_{ab}\cdot\vec{e}\right)[\mathrm{e}^{\mathrm{i}(\Delta t'-\phi(t'))}\rho_{ba}(0) - \mathrm{e}^{-\mathrm{i}(\Delta t'-\phi(t'))}\rho_{ab}(0)]\mathrm{e}^{2\gamma t'}\right\}\mathrm{e}^{-2\gamma t} \tag{12a}$$

and

$$\rho_{ab}(t) = \left\{\rho_{ab}(0) + \mathrm{i}\int_0^t \mathrm{d}t'\left(\frac{\mathcal{E}_0}{2}\vec{\mu}_{ab}\cdot\vec{e}\right)(1-2\rho_{aa}(0))\,\mathrm{e}^{\mathrm{i}(\Delta t'-\phi(t'))}\,\mathrm{e}^{(2\mathrm{i}\Omega_+ +\gamma)t'}\right\}\mathrm{e}^{-(2\mathrm{i}(\Omega_+ +\omega_0)+\gamma)t}. \tag{12b}$$

Equations (12) have terms with exponential decay (imaginary factors) but they are time-dependent, as was obtained in [23, 29], in opposition to the results of [19] (see equation (2*c*)).

Strictly speaking, geometric phases appear in adiabatic processes that happen only in the limit $T \to \infty$. However, this limit is experimentally implemented by taking $T \gg 2\pi/\omega$. The experimentally measured path-dependent phases are pretty much independent of the particular value of T, once the previous inequality is true. In equation (4.3) of [19] we have two imaginary phases: one of them ($\mathrm{e}^{-2\mathrm{Im}(\beta_-)}$) does not depend on the value of T and consequently is path-dependent, whereas the other one depends on the particular value of T. In our results (equations (10) and (11)), valid for arbitrary intensities of the classical external electric field, all the imaginary phases depend on the chosen value of T. It means that the greater the period T of $\phi(t)$, the smaller the probability that the electron will be in the upper level; also, the smaller the correlation $\rho_{ab}(t)$.

In summary, we study the nature of the imaginary phase acquired by the probability of a two-level model coupled to an external electric field with fixed frequency due to its interaction

[3] We define the time-ordering operator as

$$\mathcal{T}(\mathrm{e}^{\int_t^\tau \mathbf{A}(t')\mathrm{d}t'}) \equiv \mathbf{1} + \int_t^\tau \mathbf{A}(t_1)\,\mathrm{d}t_1 + \int_t^\tau \mathbf{A}(t_1)\,\mathrm{d}t_1\int_t^{t_1}\mathbf{A}(t_2)\,\mathrm{d}t_2 + \int_t^\tau \mathbf{A}(t_1)\,\mathrm{d}t_1\int_t^{t_1}\mathbf{A}(t_2)\,\mathrm{d}t_2\int_t^{t_2}\mathbf{A}(t_3)\,\mathrm{d}t_3 + \cdots$$

where τ can be smaller or bigger than t.

with the electromagnetic vacuum. The interaction between the quantum system (two-level system), the environment (vacuum) and the external classical electric field is taken in the electric dipole approximation and in the RWA. Garrison and Wright in [19] discussed this same model using the non-Hermitian Schrödinger equation approach, concluding that the probability has an imaginary phase that is path-dependent. In this brief report we apply the matrix density formalism to study its limit of $T \to \infty$ and from equations (10) and (11) we conclude that all imaginary phases are time-dependent and consequently depend on the chosen value of T. We are considering the situation when the effects due to dissipation and time variation of the Hamiltonian are of the same order and consequently the coherence of initial states is not preserved along the whole period T. It is not surprising that those distinct approaches (master equation and non-Hermitian Hamiltonian) give different results.

Finally we showed in the present work that even though the non-Hermitian Hamiltonian method has been a very important and useful tool in describing open systems, it should not be applied to discuss the nature (time- or path-dependent) of imaginary phases.

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